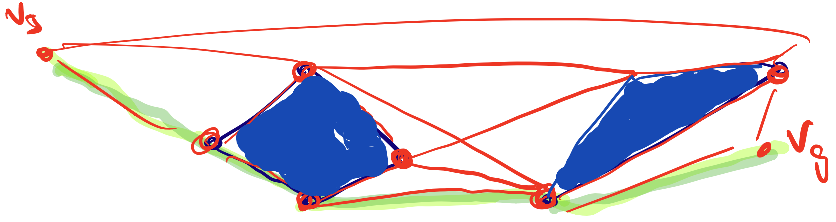
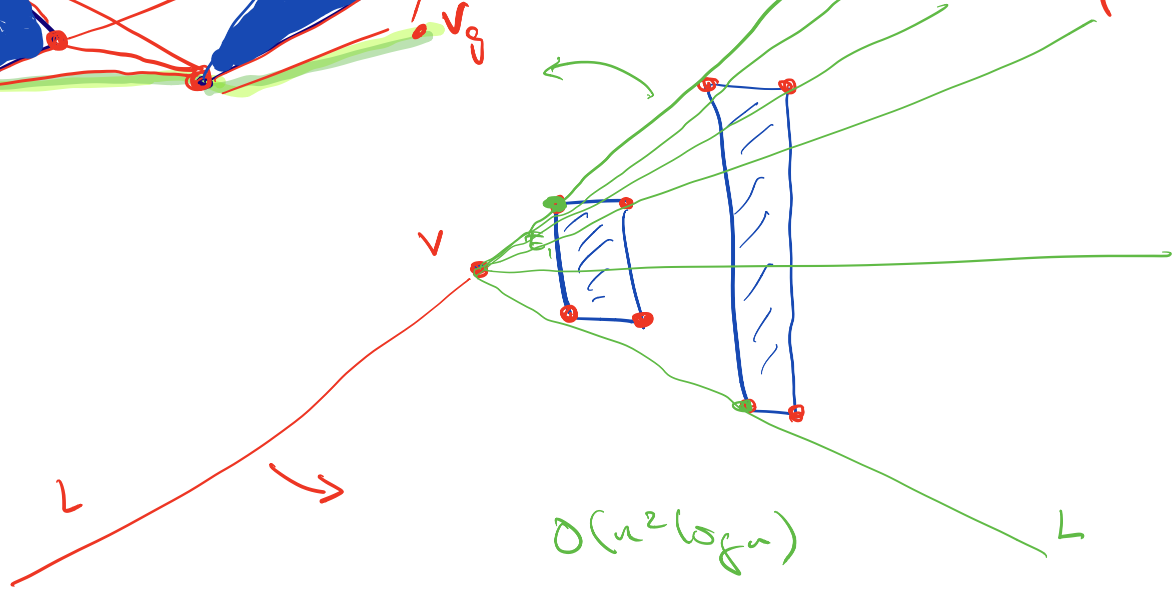
Visibility Graph: Review



→ How to construct this?

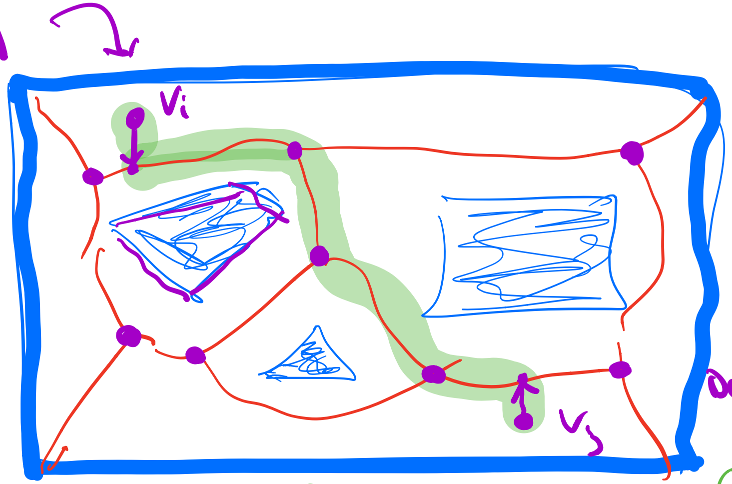
A: Naive, $O(n^3)$, n vertices

A: Better: line sweep



Generalized Voronoi Diagram $\Rightarrow$ Set of points equidistant to multiple obstacles

Accessibility



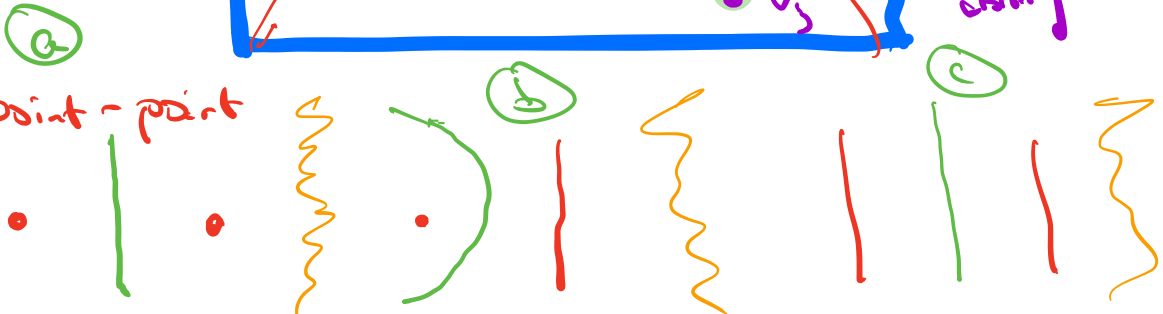
Departability

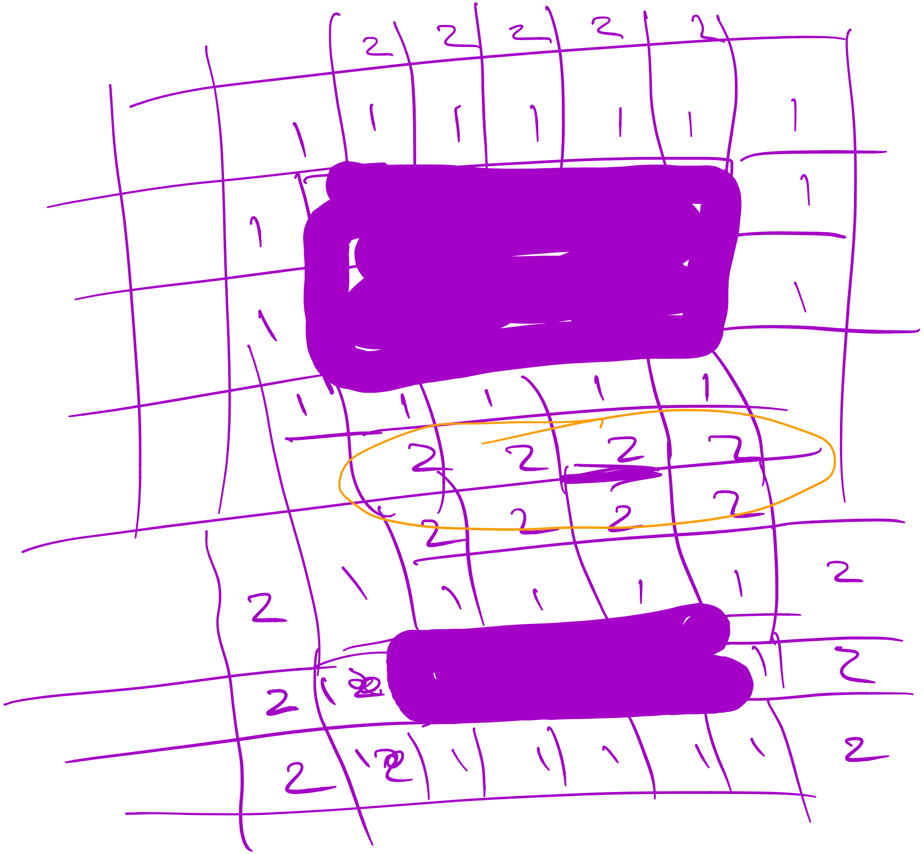
Construction

1. geometry
   - point - point
   - point - line
   - line - line

2. Wavefront on grid.

point - point





Wavefront expansion on grid

⇒ Signed Distance Function

Roadmaps: one-dimensional network of curves that "captures the connectivity" of free space.

→ Visibility Graph

Voronoi Diagram (Generalized)

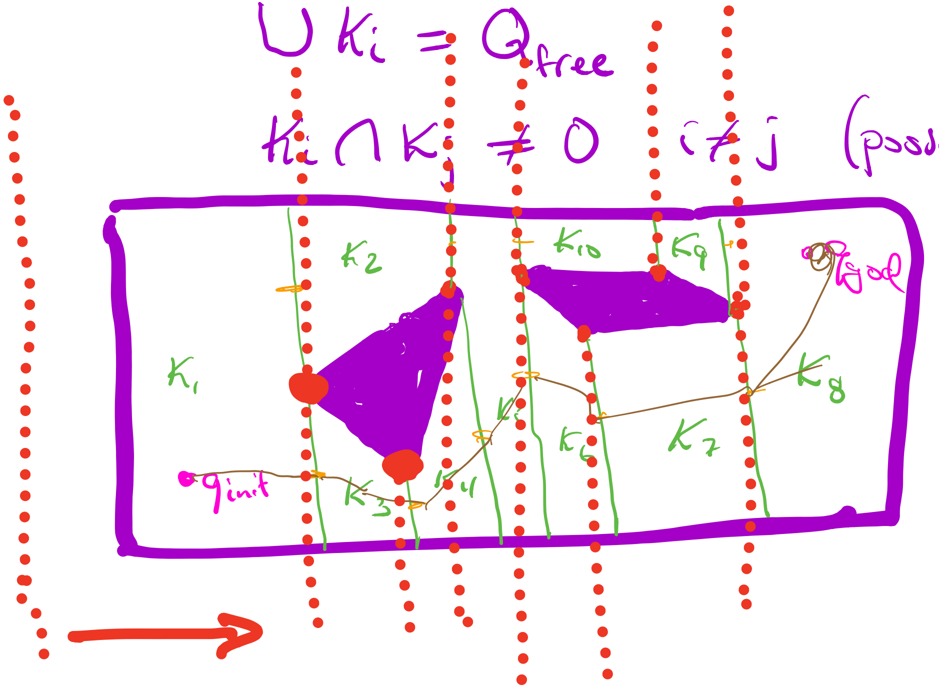
See Choset, et al.

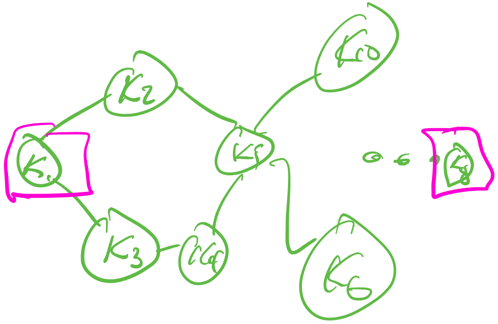
Cell Decomposition:

partition Free space into cells K_i s.t.

$\bigcup K_i = Q_{free}$

$K_i \cap K_j \neq 0 \quad i \neq j$ (possibly = boundary of K)





Artificial Potential Fields

→ For now, $Q = \mathbb{R}^n$ (no rotations)

- Consider q to be a "charged" particle $(+)$
- q_{goal} has $-$ charge
- obstacle edges/boundary has $+$ charge

$\Rightarrow$ attract to goal
repel from obstacles

More formally

Define a potential fn $U : \mathbb{R}^n \to \mathbb{R}$

s.t. $U(q_{goal})$ is min

$U(q) \to \infty$ as $q \to$ obstacle

$\to$ Then use ∇U to find a path

$$q^{i+1} = q^i - \alpha_i \nabla U(q^i)$$

} gradient descent

Typical Approach

Attracts to goal

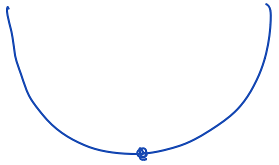
repels from obstacles

$$U(q) = U_{attr}(q) + U_{rep}(q)$$

① $U_{attr}(q) = \|q - q_{goal}\| \rightarrow \nabla U_{attr} = \dfrac{q - q_g}{\|q - q_g\|}$ unit Vector

② $U_{attr}(q) = \frac{1}{2}\|q - q_{goal}\|^2$

Parabolic Well



$$\nabla U = q - q_{goal} \checkmark$$

$$U_{rep}(q) = \begin{cases} \frac{1}{2}\left(\frac{1}{D(q)} - \frac{1}{Q^*}\right)^2 & D(q) \le Q^* \\ 0 & D(q) > Q^* \end{cases}$$

Define $D(q) = \min\limits_{q' \in \partial Q_{obst}} \|q - q'\|$

Define Q^* = distance of influence

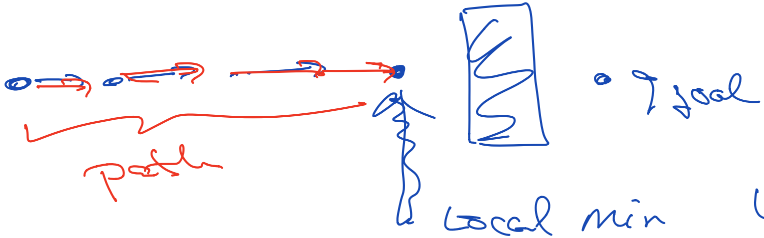
Discussion

$U \neq 0$ at goal if obstacles near by.

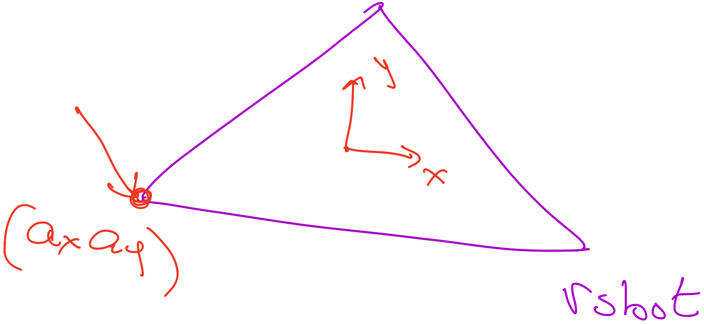
$\Rightarrow$ Local Minima are a problem

Problem for all gradient descent

(unless... convex problem)



$U_{rep} = -U_{attr}$



$$q = (x, y, \theta)$$

$$J = \left[\frac{\partial f}{d q} \right]$$

$$f(x, y, \theta) \rightarrow (a_x\ a_y)$$

Details

later ---